Journal of Physics Education and Science Volume: 2, Number 3, 2025, *Page*: 1-5

A Brief Review on Equation of Motion of Variable Mass System

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DOI:
https://doi.org/10.47134/physics.v2i3.1723
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Received: 28-04-2025
Accepted: 28-05-2025
Published: 28-06-2025

Abstract: Newton's second law states that the external force acting on a system is equal to the rate of change of its linear momentum. It is Galilean invariant when the system mass is constant. There is a misunderstanding that Newton's second law is neither a valid expression nor Galilean invariant in case of variable mass system. In a variable mass system, total system mass is a still remaining constant but remnant and ejected masses vary with time at equal rates. In case of rocket, system total mass always constant but remnant mass decreasing with time and ejected mass is increasing with time at equal rates. In the existing literature equation of motion of variable system considered as equation of motion of present remnant mass. In this work we consider total mass, remnant mass and ejected mass are three different systems and Newton's law validity and its Galilean invariance are verified for these three systems.

Keywords: Newton's Second Law, Galilean Invariance, Variable Mass System

Introduction

Laws of physics are same under Galilean transformations, in all inertial frames at non relativistic speeds. External force acting on a system of constant mass is Galilean invariant. If the system mass vary (either increase or decrease) with the time, the system is called as variable mass system. At instant t, the variable system has mass m and velocity is $\vec{v}$. Then from the Newton's second law, external force acting on the system with respect to the stationary observer is

$$\vec{F}_{ext} = m\,\frac{d\vec{v}}{dt} + \vec{v}\,\frac{dm}{dt} \tag{1}$$

If the observer is moving with velocity v_0 then,

$$\vec{F}_{ext} = m\,\frac{d(\vec{v} \pm \vec{v}_0)}{dt} + (\vec{v} \pm \vec{v}_0)\,\frac{dm}{dt} \tag{2}$$

$$\vec{F}_{ext} = m\,\frac{d\vec{v}}{dt} + (\vec{v} \pm \vec{v}_0)\,\frac{dm}{dt} \tag{3}$$

Therefore equation (1) is not Galilean invariant. This is the misconception. Because $\vec{F}_{ext}$ includes the force applied by ejected mass on remnant mass. In the literature (Alameh, 2024) (Halliday, Resnick, R., & Walker, J., 2011) (Morin, 2008) (Goldstein, 2011) Galilean invariant expression is derived by neglecting the product $\Delta m\, \Delta v$.

Methodology

In variable mass system, we can select the system in three different ways. These are remnant mass m_1, ejected mass m_2 and total mass of the system m_0. System mass m_0 is constant but m_1 and m_2 varies with time at constant rate. Mass m_1 is decreasing and m_2 is increasing with time. In this theoretical analysis Galilean invariant equations are derived for these three systems.

$$m_0 = m_1 + m_2 \tag{4}$$

$$\frac{dm_1}{dt} = -\frac{dm_2}{dt} \tag{5}$$

Results and Discussions

At instant t, mass m_1 (rocket) is moving with velocity $\vec{v}_1$, external force with respect stationary observer is given by

$$\vec{F}_1 = m_1\frac{d\vec{v}_1}{dt} + \vec{v}_1\frac{dm_1}{dt} \tag{6}$$

Similarly external force on m_2 (ejected gas jet) is given by

$$\vec{F}_2 = m_2\frac{d\vec{v}_2}{dt} + \vec{v}_2\frac{dm_2}{dt} \tag{7}$$

If the observer is moving with velocity $\vec{v}_0$

$$\vec{F}_1 = m_1\frac{d(\vec{v}_1 \pm \overrightarrow{v_0})}{dt} + (\vec{v}_1 \pm \overrightarrow{v_0})\frac{dm_1}{dt}$$

$$\vec{F}_1 = m_1\frac{d\vec{v}_1}{dt} + (\vec{v}_1 \pm \overrightarrow{v_0})\frac{dm_1}{dt} \tag{8}$$

Similarly force $$\vec{F}_2 = m_2\frac{d\vec{v}_2}{dt} + (\vec{v}_2 \pm \overrightarrow{v_0})\frac{dm_1}{dt} \tag{9}$$

Here v_2 is the velocity of the mass m_2 with respect the stationary observer. In the literature (Goldstein, 2011) (Halliday, Resnick, R., & Walker, J., 2011) (Morin, 2008) the misinterpretation is equations (8) and (9) are not Galilean invariant and hence they are not valid expressions.

A force vector diagram is shown in fig 1. From the figure external force on m_1 is (Pesce, 2020)

$$\vec{F}_1 = \vec{F}_{1s} + \vec{f}_{12} = m_1\frac{d\vec{v}_1}{dt} + \vec{v}_1\frac{dm_1}{dt} \tag{10}$$

Here $\vec{f}_{12}$ is the force applied by m_2 on m_1 and $\vec{F}_{1s}$ is the force on mass m_1 exerted by surroundings other than m_2.

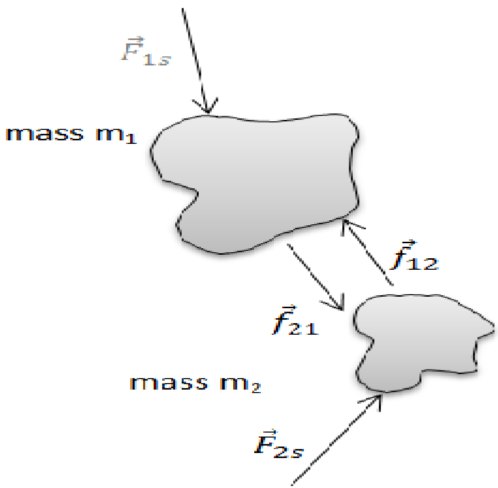

Figure 1. Forces diagram on variable mass system

Similarly force on mass m_2 is

$$\vec{F}_2 = \vec{F}_{2s} + \vec{f}_{21} = m_2 \frac{d\vec{v}_2}{dt} + \vec{v}_2 \frac{dm_2}{dt} \quad (11)$$

$\vec{f}_{12}$and $\vec{f}_{21}$ are action-reaction pairs.

$$\vec{f}_{12} = -\vec{v}_2 \frac{dm_2}{dt} \quad (12)$$

From equation (5), the external force acting on mass $m_1$is

$$\vec{F}_1 = \vec{F}_{1s} + \vec{v}_2 \frac{dm_1}{dt} = m_1 \frac{d\vec{v}_1}{dt} + \vec{v}_1 \frac{dm_1}{dt} \quad (13)$$

Equation (13) is Galilean invariant.

In the combined system of total mass m , internal forces cancel each other the external force on m_1 is

$$\vec{F}_{1s} = m_1 \frac{d\vec{v}_1}{dt} - (\vec{v}_2 - \vec{v}_1) \frac{dm_1}{dt} \quad (14)$$

Equation (14) also Galilean invariant and the thrust on m_1 is

$$\vec{F}_{thrust} = (\vec{v}_2 - \vec{v}_1) \frac{dm_1}{dt} = \vec{v}_{rel} \frac{dm_1}{dt} \quad (18)$$

Here $\vec{v}_{rel}$ is the relative velocity of m_2 with respect to m_1. Similarly from equation (7), external force acting on the m_2 is

$$\vec{F}_2 = \vec{F}_{2s} + \vec{f}_{21} = m_2 \frac{d\vec{v}_2}{dt} + \vec{v}_2 \frac{dm_2}{dt} \quad (19)$$

$$\vec{F}_2 = \vec{F}_{2s} + \vec{v}_2 \frac{dm_2}{dt} = m_2 \frac{d\vec{v}_2}{dt} + \vec{v}_2 \frac{dm_2}{dt} \quad (20)$$

External force acting on m_2 is also Galilean invariant.

The sum of equations (19) and (20) gives the total force acting on the system of total mass m.

$$\vec{\mathrm{F}} = \vec{F}_1 + \vec{F}_2 = \vec{F}_{1s} + \vec{f}_{12} + \vec{F}_{2s} + \vec{f}_{21} \quad (21)$$

Internal forces cancel each other. Therefore

$$\vec{\mathrm{F}} = \vec{F}_{1s} + \vec{F}_{2s} \quad (22)$$

$$\vec{F} = m_1 \frac{d\vec{v}_1}{dt} - (\vec{v}_2 - \vec{v}_1) \frac{dm_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} \quad (23)$$

Equation (23) is Galilean invariant, and it is a valid equation of motion of the system of total mass m.

Conclusions

The present study proves Newton's second law gives a valid and Galilean invariant expression for variable mass systems. Force on total system $\vec{F}$ force on remnant mass $\vec{F}_1$ and force on ejected mass $\vec{F}_2$, all are Galilean invariant.

This review demonstrates that Newton's second law remains valid and Galilean invariant even for systems with variable mass, such as rockets. By treating the total mass, remnant mass, and ejected mass as distinct subsystems and carefully accounting for internal and external forces, it is shown that the equations of motion derived for each subsystem retain their form under Galilean transformations. This resolves common misconceptions in the literature and reaffirms the universal applicability of Newtonian mechanics to variable mass systems.

References

Alameh, A. (2024). *Rocket Motion.* arXiv preprint arXiv:2406.00055.

DeMarini, D. M. (2022). Mutagenicity and carcinogenicity of combustion emissions are impacted more by combustor technology than by fuel composition: A brief review. *Environmental and Molecular Mutagenesis*, *63*(3), 135-150, ISSN 0893-6692, https://doi.org/10.1002/em.22475

Elliott, A.J. (2025). In-orbit system identification of a flexible satellite with variable mass using dual Unscented Kalman filters. *Acta Astronautica*, *226*, 71-86, ISSN 0094-5765, https://doi.org/10.1016/j.actaastro.2024.11.014

Gavrilov, S.N. (2024). Formal asymptotics for oscillation of a discrete mass-spring-damper system of time-varying properties, embedded into a one-dimensional medium described by the telegraph equation with variable coefficients. *Nonlinear Dynamics*, *112*(23), 20967-21002, ISSN 0924-090X, https://doi.org/10.1007/s11071-024-10154-4

Goldstein, H. (2011). *Classical Mechanics.* India: Pearson Education.

Halliday, D., Resnick, R., & Walker, J. (2011). *Fundamentals of physics* (9 ed.). New York: John Wiley & Sons, Inc.,.

Jarzębowska, E. (2024). Modeling Of Constrained Variable Mass Systems Based Upon The Boltzmann-Hamel Approach. *Proceedings of the ASME Design Engineering Technical Conference*, *9*, https://doi.org/10.1115/DETC2024-143040

Jiang, Z. (2024). A fast primal-dual algorithm via dynamical system with variable mass for linearly constrained convex optimization. *Optimization Letters*, *18*(8), 1855-1880, ISSN 1862-4472, https://doi.org/10.1007/s11590-023-02091-9

Kaczmarczyk, G. (2025). Deep-Learning Techniques Applied for State-Variables Estimation of Two-Mass System. *Energies*, *18*(3), ISSN 1996-1073, https://doi.org/10.3390/en18030568

Kalinin, Y. (2024). Solving Problems of Dynamics of Systems with Elastic Elements and Variable Masses. *Lecture Notes in Networks and Systems*, *1129*, 179-192, ISSN 2367-3370, https://doi.org/10.1007/978-3-031-70670-7_15

Kostka, Z. (2024). Dynamics Modeling Of Variable Mass Systems – A Case Study Of An Underwater Inertia Based Propelled Glider Performance. *Journal of Theoretical and Applied Mechanics*, *62*(4), 751-761, ISSN 1429-2955, https://doi.org/10.15632/jtam-pl/194505

Lam, K. F. (2024). Stability and convergence of relaxed scalar auxiliary variable schemes for Cahn-Hilliard systems with bounded mass source. *Journal of Numerical Mathematics*, *32*(3), 233-255, ISSN 1570-2820, https://doi.org/10.1515/jnma-2023-0021

Morin, D. (2008). *Introduction to classical mechanics: with problems and solutions.* Cambridge University Press.

Peraire, J. &. (2008). *Lecture L14-Variable Mass Systems: The Rocket Equation. Dynamics.*

Pesce, C. (2020). *Mechanics of variable mass systems - a historic perspective.*

Punthupeng, S. (2024). Development Of A Structural Equation Model Of The Variables Affecting The Organizational Performance Of The Mass Rapid Transit Systems Of Thailand. *Revista De Gestao Social E Ambiental*, *18*(2), ISSN 1981-982X, https://doi.org/10.24857/RGSA.V18N2-083

Suhir, E. (2022). Predictive modeling sheds useful light on burn-in testing (BIT): Brief review and recent extension. *Microelectronics Reliability*, *128*, ISSN 0026-2714, https://doi.org/10.1016/j.microrel.2021.114371

Simmonds, J. (2020). The role of agent-based modeling and multi-agent systems in flood-based hydrological problems: A brief review. *Journal of Water and Climate Change*, *11*(4), 1580-1602, ISSN 2040-2244, https://doi.org/10.2166/wcc.2019.108

Wang, B. (2024). Heuristic deepening of aero engine performance analysis model based on thermodynamic principle of variable mass system. *Energy*, *306*, ISSN 0360-5442, https://doi.org/10.1016/j.energy.2024.132481

Zhang, H. W. (2025). Dynamic Response Analysis of the Turning Variable Mass Hydrostatic Spindle System. *International Journal of Precision Engineering and Manufacturing*, ISSN 2234-7593, Https://Doi.Org/10.1007/S12541-025-01219-4

Zhang, Y. (2025). A mathematical and numerical model for variable cross-section hull girder with time-varying mass systems applied in marine vessels. *Marine Structures*, *101*, ISSN 0951-8339, https://doi.org/10.1016/j.marstruc.2024.103726