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Teaching Probability Theory and Mathematical Statistics with Practical Problems

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Abstract: This article is devoted to the role, content, and opportunities for effective teaching of elements of Probability Theory and Mathematical Statistics in the mathematics curriculum of general secondary education. It reveals the necessity of explaining the fundamental concepts of school-level probability theory—such as event, event probability, and its definitions, as well as probability calculation—through practical problems, and provides relevant problem examples. In the international PISA assessment test, questions related to this subject are also included, and students from our country have shown low performance specifically in these types of questions. This indicates that a special approach is needed for teaching this subject. In other words, in teaching the subject, it is necessary to increase students' interest through practical problems, strengthen their knowledge, and ensure that the lesson process is conducted effectively. In conclusion, it can be stated that the use of real-life, interdisciplinary, and integrated problems by the teacher in the lesson process serves to increase the effectiveness of the lesson, engage more students in the learning process, and develop their logical, statistical, and probabilistic thinking.

Keywords: Event, Event Probability, Classical, Statistical and Geometrical Definitions of Probability, Practical Problems

Introduction

Probability Theory is widely used in various fields today. Its importance in our daily lives and its role in developing analytical thinking are increasing day by day. In particular, teaching probability in schools helps students develop skills in analyzing statistical data, evaluating them, and making correct decisions in various life situations. Probability theory is taught in general secondary schools as part of the algebra curriculum. Questions related to this subject can also be found in entrance examinations to higher education institutions and in the international PISA assessment test. During PISA testing, students from our country experienced difficulties in precisely this category of questions and showed low results.

Currently, both students and teachers face various challenges in mastering this section and explaining its related topics effectively. This situation indicates the need for a special approach to teaching the subject and the necessity of methodological recommendations specifically focused on teaching probability theory in schools.

Considering this, in the present research, we have provided effective methods of teaching one of the fundamental concepts of probability theory—*the probability of an event and its definitions*—through practical problems. Additionally, we created opportunities to reinforce theoretical knowledge on the topic through real-life problem-solving examples.

Methodology

Many scholars have contributed to the development of *Probability Theory* and its establishment as a distinct scientific discipline. Among them are Blaise Pascal (one of the founders of probability theory), Pierre de Fermat (who, together with Pascal, laid the early foundations of probability theory and justified the calculation of probabilities using a combinatorial approach), Jacob Bernoulli (who proved the *Law of Large Numbers* and formulated *Bernoulli's Theorem*, which is considered one of the fundamental rules of probability), Andrey Kolmogorov (the founder of modern probability theory), Pierre-Simon Laplace (who developed the concept of *Bayesian probability*), Thomas Bayes (who introduced the fundamental rules for calculating conditional probabilities), and William Feller (who extensively explained probability theory on a large scale).

The works of these scholars have shaped probability theory and mathematical statistics into their current modern forms. Today, numerous scholars around the world continue to conduct research in this field. In particular, among international scholars contributing to the teaching of *Probability Theory* and *Mathematical Statistics* are:

- Carmen Batanero, Juan D. Godino, and Rafael Roa (2014), *Training Teachers to Teach Probability*,
- Feng Wang and Xiapping Xu, *Discussion on the Teaching Method of Probability Theory and Mathematical Statistics*,
- Renalyn De Mesa Mender, *Mathematics Teaching Strategy in Statistics and Probability*,
- Shuai Liu, Renyong Guo, et al., *An Effective Teaching Method of the Course "Probability Theory and Mathematical Statistics" in Higher Education by Formative Evaluation*,
- Murad Barakayev, Kamola Turgunova, et al., *Methodology of Teaching Theory of Probability and Elements of Mathematical Statistics with the Help of Practical Problems*,
- Wenyao Xiong, *Research on the Probability Theory and Mathematical Statistics Teaching*.

These studies present valuable insights into the effective teaching of the subject.

As L. Hetmanenko noted in his research, *"Mathematics, with its strict logic and abstract concepts, often worries students and alienates them. However, modern teaching methods based on actively involving students in the learning process can significantly improve this perception."* Interactive learning methods engage students more actively in learning. According to T. Peterson and M. Thomas (2021), *"The use of interactive games and simulations in teaching probability concepts increases students' interest in the topic and helps them develop deeper understanding."* [9]. Such methods allow students to comprehend abstract ideas more easily by connecting them to real-life scenarios, and justify the need for practical approaches to teaching probability concepts.

In order to achieve the objectives of this research and address the stated tasks, the following methods were applied:

Literature Analysis

a comprehensive review of existing literature, including scientific articles, prior studies, books, and other publications relevant to the topic in the context of mathematics education. This method provides an overview of the current results, trends, methodologies, and research in the field.

Pedagogical Practice Analysis

examining the experience of mathematics teachers by observing classroom interactions between teachers and students, analyzing instructional materials, and conducting interviews.

The integration of these methods enables a multifaceted examination of learning probability through interdisciplinary teaching, and assists in developing recommendations for optimizing and enhancing the effectiveness of the educational process.

Result and Discussion

The expected outcomes in teaching *Probability Theory* and *Statistics* are not only related to the age of learners or the scope of the topic, but also significantly depend on the instructional approach. In the recent past, the teaching of probability and statistics in schools relied heavily on a formula-based approach, which led to a decline in both students' mastery of the subject and their interest in it.

Teaching through practical problems is an innovative strategy that enhances students' engagement and shifts them from passive learning to active exploration. These methods support the development of essential 21st-century skills such as collaboration, critical thinking, problem-solving, and adaptability. Before teaching the probability of events using practical problems, let us define probability: The likelihood of a particular event successfully occurring is called the *probability of the event* and is denoted by P [10]. Below are several definitions of event probability:

Classical Definition of Probability. If an experiment has $\boldsymbol{n}$ equally likely and mutually exclusive outcomes, and $\boldsymbol{m}$ of them are favorable for event $\boldsymbol{A}$, then the probability of event $\boldsymbol{A}$ occurring is defined by the ratio:

$$P(A)=\frac{k}{n}$$

Problem: A die is rolled once. Find the probability of getting an odd number.

Solution: Let event A be "getting an odd number." There are three favorable outcomes: 1, 3, and 5. So, $m = 3$. The total number of possible outcomes is $n = 6$.

$$P(A)=\frac{3}{6}=\frac{1}{2}$$

Statistical Definition of Probability. The ***relative frequency*** of an event $\boldsymbol{A}$ is the ratio of the number of times $\boldsymbol{k}$ the event occurs in an experiment to the total number of trials $\boldsymbol{n}$, denoted as:

$$W(A) = \frac{k}{n}$$

If the number of trials is sufficiently large and the relative frequency of event A stabilizes around a constant value, then this value is called the *statistical probability* of event A. In that case:

W(A)=P(A)

Problem: In a coin-tossing experiment, what is the probability of the coin landing on the "heads" side?

Solution: Let event A be "landing on heads." There are two possible outcomes in the experiment: heads or tails, so $n = 2$.

$$W(A) = \frac{1}{2}$$

According to the definition, if this experiment is repeated a sufficiently large number of times, then W(A)=P(A).

Let us now examine how to study these concepts using interactive teaching methods. First, we outline a procedure for conducting these methods.

Practice Problems for Reinforcement

1. A box contains 10 balls: 7 black and 3 white. One ball is randomly drawn. What is the probability that it is:
2. a. white?
   b. black?

   Answer: a) 3/10; b) 7/10
3. One letter is randomly selected from the word "EHTIMOLLIK".
   a. What is the probability that the letter is "A"?
   b. What is the probability that it is a vowel?

   Answer: a) 0; b) 4/10
4. Three coins are tossed. What is the probability that exactly two of them land on heads?

   Answer: 3/8
5. A die is rolled once.
   a. What is the probability of getting the number 4?
   b. What is the probability of getting a number greater than 4?

   Answer: a) 1/6; b) 1/3
6. A shooter has a relative hit frequency of 0.6. If the shooter missed the target 12 times, how many total shots were fired?

   Answer: 30

Geometric Probability. Geometric probability is used in experiments where the number of elementary outcomes is infinite (either countably or uncountably infinite). The concept of geometric probability can be explained through the following example:

Suppose a point is randomly thrown into a region G, that is, the point can fall anywhere within the region G with equal probability. The probability that the point falls into a subregion g⊂G \subset G is proportional to the measure (length, area, volume, etc.) of the subregion gg, regardless of its location or shape. Thus, the probability that the point lands in the region g is:

$$P(g)=\frac{g \text{ area measurement}}{G \text{ area measurement}}$$

Problem. A circle is inscribed in a square with side length aa. Find the probability that a randomly chosen point within the square falls inside the circle.

Solution:

According to the problem, G is a square with side length a, and g is a circle of radius a/2 inscribed within the square. Since both shapes are considered in the plane, their measures are interpreted as area. Thus, the required probability is:

$$\mathrm{P(g)}=\frac{g \text{ area measurement}}{G \text{ area measurement}}=\frac{\text{surface } g}{\text{surface } G}=\frac{\pi(a/2)^2}{a^2}=\frac{\pi}{4}$$

Practice Problems

After a storm, a telephone line is broken somewhere between kilometers 40 and 70. What is the probability that the break occurred between kilometers 50 and 55?

Answer: 1/6

A small circle with radius rr is placed inside a larger circle with radius RR. What is the probability that a randomly selected point inside the larger circle also lies within the smaller circle?

Answer: $p=r^2/R^2$

Sample Problems with Solutions

Problem 1. A box contains 3 blue, 4 yellow, and 5 red balls. One ball is randomly selected. What is the probability that the ball is:

1) blue? 2) yellow? 3) red?

Solution:

1) $P(A)=\frac{k}{n}=\frac{3}{12}$ 2) $P(B)=\frac{k}{n}=\frac{4}{12}$ 3) $P(C)=\frac{k}{n}=\frac{5}{12}$

Problem 2. In a lottery, there are 1,000 tickets, and 20 of them are winning tickets. One ticket is purchased. What is the probability that the ticket is: winning? 2) not winning?

Solution:

$P(A)=\frac{k}{n}=\frac{20}{1000}$, $P(B)=\frac{k}{n}=\frac{980}{1000}$

Problem 3. A student has not prepared for 3 out of 30 possible exam questions. What is the probability that the student will receive a question they are prepared for?

Solution:

Favorable outcomes: k=27, Total outcomes: n=30

$$P(A)=\frac{k}{n}=\frac{27}{30}$$

Problem 4. A coin is tossed 6 times and lands on heads each time. What is the probability that it will land on heads on the 7th toss?

Solution:

Since each coin toss is an independent event and has only two outcomes (heads or tails), the probability remains constant for each trial:

$$P(A)=\frac{k}{n}=\frac{1}{2}$$

Conclusion

Teaching through practical problems transforms the educational process from passive knowledge transfer into an active, collaborative learning environment that enhances student engagement. Using such approaches, teachers can apply various strategies aligned with lesson objectives to deliver in-depth knowledge, foster conceptual understanding, and develop essential life skills among students. Teaching the probability of events through practical and interdisciplinary problems creates an effective and engaging learning atmosphere.

This method enables students not only to gain theoretical knowledge about probability but also to apply that knowledge in real-life contexts. In everyday situations, we often hear phrases like "this is more likely" or "that is less likely," which reflect probabilistic reasoning. Therefore, learning to calculate probabilities helps individuals make better decisions in various life scenarios. Furthermore, opportunities to work with modern technologies and real-world data contribute to the development of students' analytical thinking and enhance their logical and statistical reasoning.

Several challenges arise in teaching elements of *Probability Theory* and *Mathematical Statistics*, and the following recommendations are proposed to address these issues:

- New, adapted textbooks are required to facilitate the integration of knowledge across disciplines;
- Joint efforts from all educational departments are necessary to support teachers in applying integrated teaching methodologies;
- It is essential to improve the pedagogical skills and professional training of mathematics teachers.

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