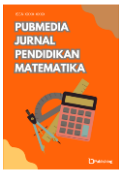

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Estimating The Hazard Function for A Mixed Distribution Using the Genetic Algorithm with A Practical Application

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Abstract: In this research, we presented the EKIW (Exponentiated Kumaraswamy Inverse Weibull) Distribution, which is a flexible distribution used in survival and reliability analysis. This distribution includes parameters (α, β, λ, η, θ). Statistical properties such as the hazard function, the quantile function, and the entropy measure were derived for this distribution. The research aims to estimate the hazard function based on one of the artificial intelligence algorithms, which is the genetic algorithm. It was compared with the maximum likelihood method. To prove the applicability of this distribution and which of the two methods is better, sample sizes (n=5, 15, 30, 80, 150) were generated using the comparison criterion, which is the integral mean square error (IMSE). The research also included a practical application for lung cancer patients obtained from the Medical City Hospital in 2025. The results showed that the hazard function's capabilities increased with increasing time of infection for the group of lung cancer patients under study. This is consistent with the theoretical properties of this function, as it is an increasing monotonic function.

Keywords: EKIW Distribution, Kumaraswamy Distribution, Weibull Distribution, Properties, Maximum Likelihood Method, Genetic Algorithm, IMSE, Hazard Function

Introduction

The inverse Weibull distribution is typically employed in the domains of reliability analysis and biological research. Over an extensive period, scholars have been engaged in the development of various extensions and adaptations of the inverse Weibull distribution. Recently, proposed an extension of the inverse Weibull distribution identified as the beta generalized inverse Weibull distribution. delineated a three-parameter generalized inverse Weibull distribution characterized by a decreasing and unimodal failure rate. introduced the modified inverse Weibull distribution and examined its numerous properties. presented and analyzed a four-parameter distribution referred to as the beta inverse Weibull distribution, while also introduced and investigated a four-parameter inverse Weibull distribution. The cumulative distribution function (CDF) and probability density function (PDF) corresponding to the inverse Weibull distribution are articulated by

$$G(x;\alpha,\beta) = \exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right] \qquad (1.1)$$

and,

$$g(x;\alpha,\beta) = \beta\alpha^{\beta}x^{-(\beta+1)}\exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right] \quad (1.2)$$

respectively, where $x > 0$; $\alpha > 0$ and $\beta > 0$.

This manuscript introduces a novel statistical distribution referred to as the Kumaraswamy inverse-exponential Weibull distribution. This distribution encompasses, as particular instances, the inverse exponential distribution, the inverse Weibull split, the inverse Rayleigh split, and the inverse-exponential Weibull split. A comprehensive examination of various attributes of the newly proposed model is conducted, thoroughly illustrated for this scholarly work, and applied to an empirical dataset.

Research Problem

The maximum likelihood (MLE) method may face problems such as convergence to non-optimal local points or numerical instability. The proposed solution is to employ a genetic algorithm (GA) with MLE to improve the accuracy and efficiency of the estimation.

Kumaraswamy Distribution

He introduced a two-parameter distribution model, named the Kumaraswamy distribution, which is given by his cumulative distribution function.

$$F(x;\lambda;\eta) = 1-(1-x^{\lambda})^{\eta} \quad (3.1)$$

where $0 < x < 1$; $\lambda > 0$ and $\eta > 0$. The CDF corresponding to (3.1) is given by:

$$f(x;\lambda;\eta) = \lambda\eta x^{\lambda-1}(1-x^{\lambda})^{\eta-1} \quad (3.2)$$

The Kumaraswamy distribution has been identified as a useful alternative to the beta distribution, due to the similarity of their basic shape properties (unimodal, increasing, decreasing, monotonic, or constant). However, the likelihood function (PDF) introduced in (3.2) does not include any incomplete beta function ratio and is considered unsolvable due to its algebraic properties. A long time ago, proposed an alternative to the Kumaraswamy distribution, called the exponential Kumaraswamy distribution. The likelihood function (CDF) and the likelihood state (PDF) of the exponential Kumaraswamy distribution are given by:

$$F(x;\lambda;\eta;\theta) = [1-(1-x^{\lambda})^{\eta}]^{\theta} \quad (3.3)$$

And,

$$f(x;\lambda;\eta;\theta) = \lambda\theta\eta x^{\lambda-1}(1-x^{\lambda})^{\eta-1}[1-(1-x^{\lambda})^{\eta}]^{\theta-1} \qquad (3.4)$$

, where:
$0 < x < 1$; λ, η, $\theta > 0$;

Let $G(x)$ be the CDF of a random variable X. The CDF of a generalized class of distributions is given by

$$F(x;\lambda;\eta;\theta) = [1-(1-G(x)^{\lambda})^{\eta}]^{\theta} \qquad (3.5)$$

The corresponding PDF to (3.5) is:

$$f(x;\lambda;\eta;\theta) = \lambda\theta\eta g(x)G(x)^{\lambda-1}(1-G(x)^{\lambda})^{\eta-1}[1-(1-(1-G(x)^{\lambda})^{\eta}]^{\theta-1} \qquad (3.6)$$

We used the cumulative distribution function of the Dagom distribution in (3.6) to propose the exponential Kumaraswamy -Dagom distribution. The generalization (3.6) can be used to propose other quotients of the exponential Kumaraswamy quotient.

Substituting (1.1) into (3.5), we obtain a new distribution, called the exponential Kumaraswamy Weibull (EKIW), with a cumulative distribution function given by the following formula:

$$F(x;\alpha;\beta;\lambda;\eta;\theta) = [1-(1-\exp[-\lambda(\alpha/x)^{\beta}])^{\eta}]^{\theta} \qquad (3.7)$$

The pdf is:

$$f(x;\alpha;\beta;\lambda;\eta;\theta) = \beta\lambda\theta\eta\alpha^{\beta}\beta\ x^{-(\beta+1)}\ \exp[-\lambda(\alpha/x)^{\beta}](1-\exp[-\lambda(\alpha/x)^{\beta}])^{\eta-1}$$
$$*[1-(1-\exp[-\lambda(\alpha/x)^{\beta}])^{\eta}]^{\theta-1} \qquad (3.8)$$

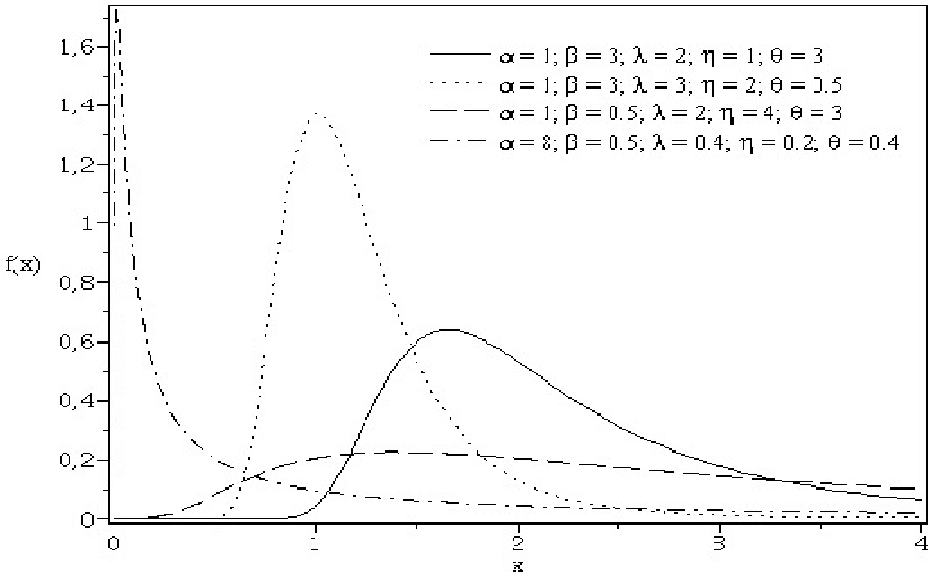

Figure 1. This figure shows the probability function of the EKIW distribution when changing the values of the parameters α, β, α, η, θ.

Statistical Properties of the EKIW Distribution

Expansions for the cumulative and density functions for EKIW:

When is a real number greater than zero and not an integer, we get the following power series?

$$(1-\omega)^{\theta-1} = \sum_{j=0}^{\infty} \frac{(-1)^j\,\Gamma(\theta)\,\omega^j}{\Gamma(\theta-j)\,j!} \qquad (4.1)$$

where $|\omega| < 1$ and $\Gamma(\cdot)$ is the gamma function :

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1}\, e^{-t} dt \qquad (4.2)$$

Using the power series (4.1) in Equation (3.7), we can write

$$F(x;\alpha;\beta;\lambda;\eta;\theta) = \theta \sum_{j=0}^{\infty} \frac{(-1)^j\,\Gamma(\theta)\,\omega^j}{\Gamma(\theta-j)\,j!} \left(1-\exp[-\lambda(\alpha/x)^{\beta}]\right)^{j\eta} \qquad (4.3)$$

When is an integer, the index in the preceding sums terminates at. By applying once more the power series from Equation (4.3), we can rewrite (3.7) for any real non-integer as

$$F(x;\alpha,\beta,\lambda,\eta,\theta) = \theta\eta \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{(-1)^{j+k}\Gamma(\theta)\Gamma(j\eta)}{\Gamma(\theta-j+1)\Gamma(j\eta-k+1)(j-1)!k!} \exp\left[-\lambda k\left(\frac{\alpha}{x}\right)^{\beta}\right] \qquad (4.4)$$

follows:

By using the power series (4.1) in the Equation (3.8), we obtain:

$$f(x;\alpha,\beta,\lambda,\eta,\theta)=\frac{\beta\lambda\theta\eta\alpha^{\beta}}{x^{\beta+1}}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{j+k}\Gamma(\theta)\Gamma[\eta(j+1)]}{\Gamma(\theta-j)\Gamma[\eta(j+1)-k]\,j!k!}\exp\left[-\lambda\,(k+1)\left(\frac{\alpha}{x}\right)^{\beta}\right] \tag{4.5}$$

Risk and reverse Risk function:

For a continuous distribution with PDF $f\,(x)$ and CDF $F(x)$, the Risk function is defined by:

$$h(x)=\lim_{\Delta x\to 0}\frac{P(X<x+\Delta x|X>x)}{\Delta x}\ /=\frac{f\ (x)}{1-F(x)} \tag{4.6}$$

The risk function is an important quantity characterizing life phenomena. For the EKIW distribution, the risk rate function defines:

$$h(x;\alpha,\beta,\lambda,\eta,\theta)=\frac{\beta\lambda\theta\eta\alpha^{\beta}x^{-(\beta+1)}\exp\left[-\lambda\left(\frac{\alpha}{x}\right)^{\beta}\right]\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x}\right)^{\beta}\right]\right)^{\eta-1}}{1-\left[1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x}\right)^{\beta}\right]\right)^{\eta}\right]^{\theta}} \times\left[1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x}\right)^{\beta}\right]\right)^{\eta}\right]^{\theta-1} \tag{4.7}$$

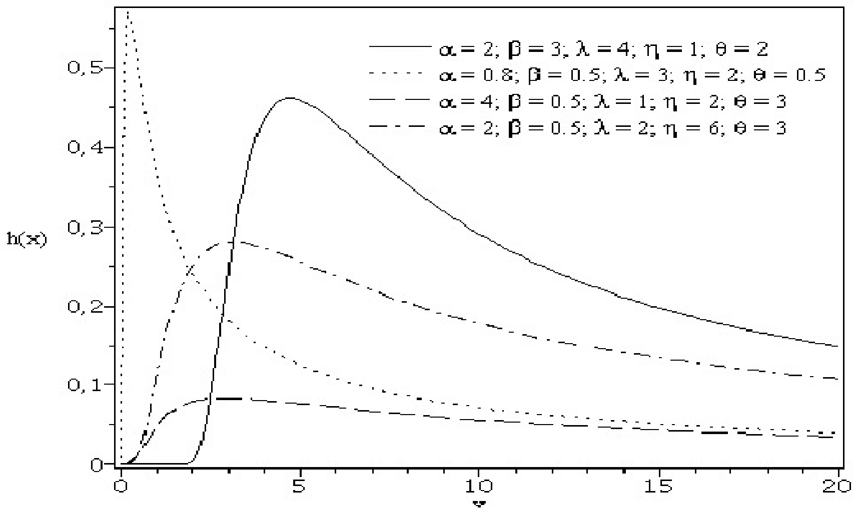

Figure 2. This figure shows the risk function of the EKIW distribution when changing the values of the parameters α, β, α, η, θ.

The reverse risk function of the EKIW distribution is:

$$h(x;\alpha,\beta,\lambda,\eta,\theta)=\frac{\beta\lambda\theta\eta\alpha^{\beta}x^{-(\beta+1)}\exp\left[-\lambda\left(\frac{\alpha}{x}\right)^{\beta}\right]\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x}\right)^{\beta}\right]\right)^{\eta-1}}{\left[1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x}\right)^{\beta}\right]\right)^{\eta}\right]^{\theta}}$$
$$\times\left[1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x}\right)^{\beta}\right]\right)^{\eta}\right]^{\theta-1} \quad (4.8)$$

Quantile function of EKIW

The quantile function, for the EKIW distribution is define by:

$$Q(p)=\frac{\alpha}{\{-\log[1-(1-p^{1/\theta})^{1/\eta}]^{1/\lambda}\}^{1/\beta}} \quad (4.9)$$

When:

$Q(p)$, $0<p<1$

The median of the distribution is obtained by using p = 0:5 in (4.9). The random sample can also be easily generated from (4.9) by using p as uniform random number.

Entropy

The entropy of a random variable quantifies the degree of uncertainty or variability. This concept plays a central role across various scientific disciplines, including communication theory, physics, and probability. Among the most well-known entropy measures is Rényi entropy. For a random variable with probability density function, the Rényi entropy is defined as:

$$HR(\nu)=\frac{1}{1-\nu}\log\left[\int f^{\nu}(x)dx\right] \quad (4.10)$$

where $\nu>0$ and $\nu\neq 1$. Using (4.10), Renyi entropy of EKIW distribution is define by

$$H_R(\nu)=\frac{\nu}{1-\nu}(\log\beta+\log\lambda+\log\theta+\log\eta+\beta\log\alpha)+\frac{1-\nu(\beta+1)}{1-\nu}\log\alpha-\log\beta$$
$$+\frac{1}{1-\nu}\log\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{j+k}\Gamma[\nu(\theta-1)+1]\Gamma[\eta j+\nu(\eta-1)+1][\lambda(\nu+k)]^{[1-\nu(\beta+1)]/\beta}}{\Gamma[\nu(\theta-1)+1-j]\Gamma[j\eta+\nu(\eta-1)+1-k]\,j!k!}$$
$$\times\Gamma\left[\frac{\nu(\beta+1)-1}{\beta}\right]$$

(4.11)

Parameter Estimation

Maximum Likelihood Estimation (MLE)

This method is considered one of the most effective estimation methods, as it has good properties, including optimal representation and consistency in some cases. Suppose we have several observations from a triangular EKIW distribution. Then, we obtain the likelihood function as follows:

Now, let's estimate using the maximum likelihood method. The logarithmic probability of a direct sample *x*1*; ….;xn* from the EKIW distribution, given by Equation (3.8), is:

$$\log L(\alpha,\beta,\lambda,\eta,\theta)=n\log\beta+n\log\lambda+n\log\theta+n\log\eta+n\beta\log\alpha-(\beta+1)\sum_{i=1}^{n}\log x_i$$
$$-\lambda\sum_{i=1}^{n}\left(\frac{\alpha}{x_i}\right)^{\beta}+(\eta-1)\sum_{i=1}^{n}\log\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)$$
$$+(\theta-1)\sum_{i=1}^{n}\log\left[1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{\eta}\right]$$

(5.1)

By taking the derivative of the log-likelihood with respect to and in turn, and equating each to zero, we obtain:

$$\frac{\partial \log L}{\partial \alpha} = \frac{\beta\lambda\eta(1-\theta)}{\alpha}\sum_{i=1}^{n}\frac{\left(\frac{\alpha}{x_i}\right)^{\beta}\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{\eta-1}}{1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{\eta}}$$

$$+\frac{n\beta}{\alpha}-\frac{\lambda}{\alpha}\sum_{i=1}^{n}\left(\frac{\alpha}{x_i}\right)^{\beta}-\beta\lambda(\eta-1)\alpha^{\beta-1}\sum_{i=1}^{n}\frac{x_i^{-\beta}}{1-\exp\left[\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]}=0 \quad (5.2)$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta}+n\log\alpha-\sum_{i=1}^{n}\log x_i+\lambda\sum_{i=1}^{n}\left(\frac{\alpha}{x_i}\right)^{\beta}\log\left(\frac{\alpha}{x_i}\right)+(1-\eta)\sum_{i=1}^{n}\frac{\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\log\left(\frac{\alpha}{x_i}\right)}{1-\exp\left[\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]}$$

$$-\lambda\eta(\theta-1)\sum_{i=1}^{n}\frac{\left(\frac{\alpha}{x_i}\right)^{\beta}\log\left(\frac{\alpha}{x_i}\right)\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{\eta-1}}{1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{\eta}}=0 \quad (5.3)$$

$$\frac{\partial \log L}{\partial \lambda} = \eta(1-\theta)\sum_{i=1}^{n}\frac{\left(\frac{\alpha}{x_i}\right)^{\beta}\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{\eta-1}}{1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{\eta}}$$

$$+\frac{n}{\lambda}+\sum_{i=1}^{n}\left(\frac{\alpha}{x_i}\right)^{\beta}-(\eta-1)\alpha^{\beta}\sum_{i=1}^{n}\frac{x_i^{-\beta}}{1-\exp\left[\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]}=0 \quad (5.4)$$

$$\frac{\partial \log L}{\partial \eta} = \frac{n}{\eta}+(\theta-1)\sum_{i=1}^{n}\frac{\log\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)}{1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{-\eta}}$$

$$+\sum_{i=1}^{n}\log\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)=0 \quad (5.5)$$

$$\frac{\partial \log L}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \log\left[1-\left(1-\exp\left[-\lambda\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)^{\eta}\right] = 0 \quad (5.6)$$

Genetic algorithm method

A genetic algorithm is a random search method that addresses a problem to reach the best possible results. It revolves around evolutionary techniques based on Dar's theory of evolution, which states that the fittest survive by imitating the work of nature by preserving the good qualities present in the parents' generation and transferring them to the offspring's generation. Its goal is to obtain immediate offspring that have the best qualities of the parents at the very least.

Genetic Algorithm Stages:

Although there are some differences in the implementation of genetic algorithms depending on the branches of evolutionary techniques, they generally share the following basic stages:

1. Initial Population Preparation: A set of chromosomes (the search space) is randomly generated, with each chromosome representing a potential solution to the problem.
2. Generation of the First Generation: The initial generation is formed by generating chromosomes according to the nature of the problem and the desired population size.
3. Objective Function and Evaluation: Chromosomes are evaluated using an objective function (which may be maximizing or minimizing) that assigns each chromosome a value that expresses how close it is to the optimal solution.
4. Generation of New Generations: The most efficient individuals are selected according to the "survival of the fittest" principle, and then crossover and mutation are applied to generate the next generation.
5. Iteration of the Process: These stages are repeated to generate successive generations until a pre-defined stopping criterion is met.
6. Testing: We test the solution by determining whether or not a stopping condition is met. When this condition is met, the genetic algorithm stops, and the new solution benefits from the previous generation's formation.
7. Stopping Criterion: Successive generations continue to be formed to improve the solution examples until the stopping condition is met. This condition is based on the genetic algorithm's stopping metric (the optimal solution). This metric varies depending on the problem to be addressed.
8. Termination: The genetic algorithm terminates when one of the following factors is met:

- Finding the optimal solution
- Reaching the desired number of generations
- Providing a specific value, such as calculating the cost of production
- Falling into a local minimum value and not being able to exceed it

Applying the stages of the genetic algorithm in the binary EKIW distribution:

We apply the stages of the genetic algorithm in the objective function equation for each method to find estimates of the parameters of the binary EKIW distribution according to the following:

- Initialization: Forming the chromosome through the values of $\beta_p$that form the genes of the chromosome and that (P=0,1,2,...,p) are within the real numbers.
- Initialization: Creating the initial generation by finding an initial value for the genes with the random values of the other set of constraints.
- Objective function: The chromosome is evaluated in terms of efficiency to reach the optimal solution by determining the value of β_p Perform the testing process for the chromosome that has a small objective function value by choosing the highest probability for it and finding the evaluation function for it through the following equation

$$Fitness\ Function = \frac{1}{1 + objective\ function}$$

Through the evaluation function formula, we can find the probability of this function (the best values) according to the following mathematical formula:

$$C_{(i)} = \frac{f(i)}{\sum_{i=1}^{n} f(i)} \quad (5.7)$$

Since:

$C_{(i)}$:represents the probability of individual i

$f(i)$:Evaluation function of individual i

n: represents the size of observations

In this step, chromosomes that are good in their characteristics are hybridized by mating between each two chromosomes, and one of its criteria is applied, which is organized hybridization based on the probability of hybridization Pc, and this value is compared with the value of the genes for the two chromosomes (parents) to form the new generation of children and the exchange occurs when the value of the gene is greater or equal to the probability value.

The last step that the chromosomes can go through is the mutation process and it also depends on the probability value (Pm) for the parameters of replacing randomly selected genes with a new value that we also obtained randomly according to the following formula:

Total genes = Number of genes in the chromosome * Number of population

The following diagram represents the application of the stages of the genetic algorithm for distribution (EKIW):

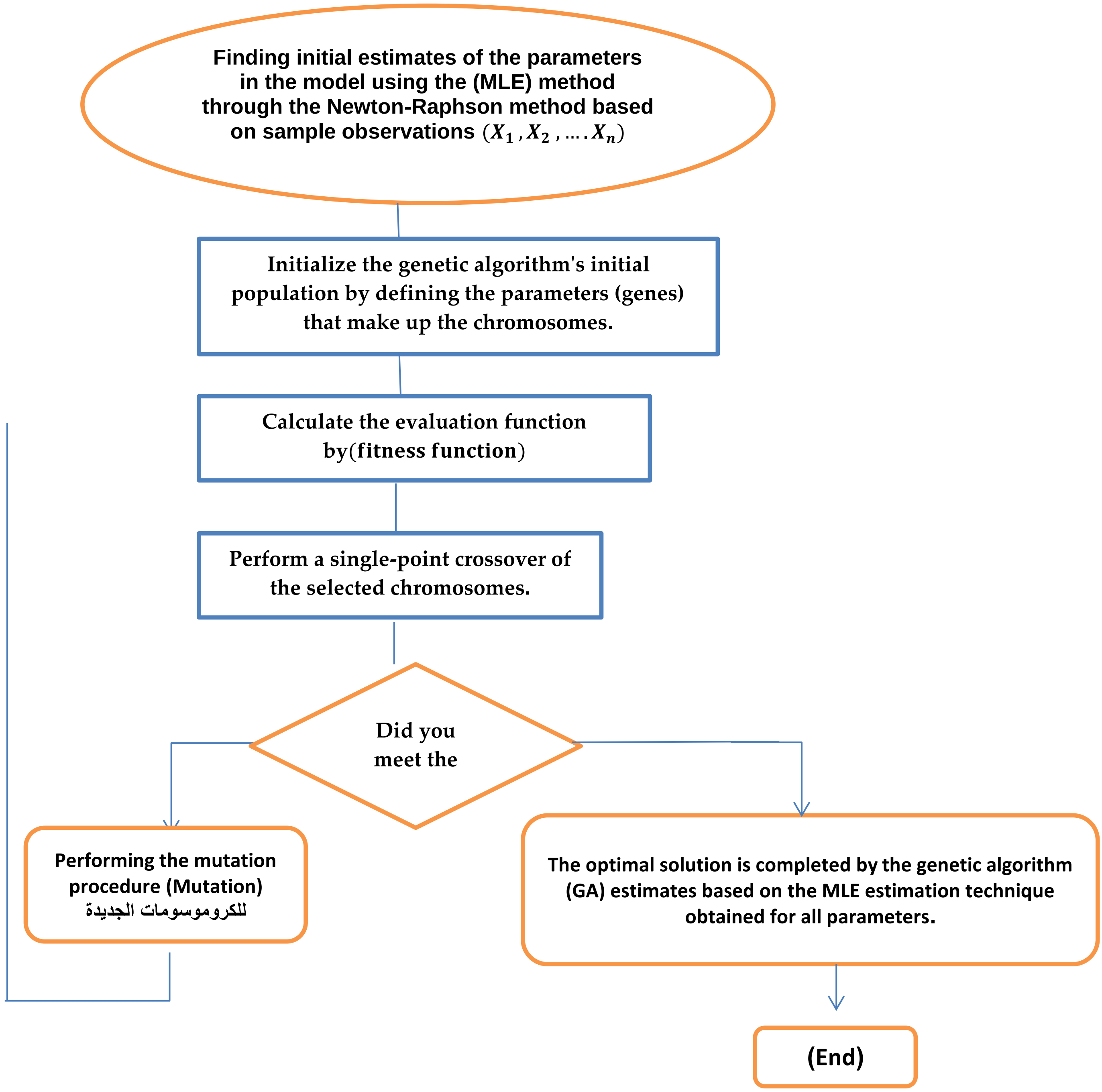

Figure 3. shows the flowchart of the genetic algorithm based on the MLE estimation technique

Experimental Aspect

The Monte Carlo simulation method was adopted to compare different estimation methods. This method is characterized by its flexibility and ability to reduce costs by taking into account varying sample sizes and different values of distribution parameters, with the possibility of repeating the experiment each time. It also allows data to be generated without the need for real data, while maintaining the required level of accuracy. Its basic steps can be summarized as follows:

1. Determining default values: Five sample sizes were chosen (5, 15, 30, 80, and 150). Default values were also used for the two parameters, as follows:
2. Data Generation :The random variable was generated using the inverse transformation method of a function as follows:

$$Q(p) = \frac{\alpha}{\{-\log[1-(1-p^{1/\theta})^{1/\eta}]^{1/\lambda}\}^{1/\beta}} \quad (6.1)$$

3. Arrange the generated data in ascending order: $X_1 < X_2 < \cdots < X_i$
4. Solve the equations obtained numerically.
5. The best method was determined using the IMSE comparison measure when estimating the risk function.

$$IMSE(\hat{h}(x)) = \frac{1}{r}\sum_{i=1}^{r}\left[\frac{1}{n_t}\sum_{j=1}^{n_t}\left(\hat{h}_i(x_j) - h(x_j)\right)^2\right] \quad (6.2)$$

Where:

r :Number of times the simulation was repeated, (5000) times in this study.

n_t: Sample size (number of observations) generated in each iteration.

$h(x_j)$: Risk function according to initial values and respectively.

Table 1. Represents the integrated mean square error (IMSE) values of the Risk function estimate for all estimation methods and sample sizes.

model	n	MLE	$G.E$	best
α=1.0,θ=0.8 β=0.8 ,λ=1.0, η =1.5	5	4.71E-08	2.87E-11	$G.E$
	15	8.33E-09	5.78E-11	$G.E$
	30	7.10E-09	2.11E-10	$G.E$
	80	2.81E-09	2.40E-10	$G.E$
	150	6.99E-12	1.28E-10	MLE
α=1.0,θ=1.5 λ=1.0, β=2.0 η =1.0	5	1.23E-08	1.81E-11	$G.E$
	15	6.58E-09	7.32E-11	$G.E$
	30	2.84E-09	1.01E-10	$G.E$
	80	2.87E-09	2.80E-10	$G.E$
	150	5.61E-11	1.20E-10	MLE
α=1.2,θ=0.6 λ=0.6, β=1.5 η =2.5	5	2.75E-07	1.07E-08	$G.E$
	15	6.48E-07	4.17E-09	$G.E$
	30	1.07E-08	1.92E-09	$G.E$
	80	1.79E-08	1.64E-09	$G.E$
	150	3.19E-10	1.35E-09	MLE
α=0.8,θ=1.2 λ=1.8, β=1.2 η =3.0	5	1.81E-08	5.50E-12	$G.E$
	15	3.86E-09	1.45E-11	$G.E$
	30	3.38E-09	5.88E-11	$G.E$
	80	1.95E-09	8.79E-11	$G.E$
	150	9.01E-10	8.73E-11	$G.E$

Table 1 shows the values of the integral mean squared error (IMSE) for the estimators of the risk function using the MLE and the genetic algorithm method, as follows:

1. We note that the genetic algorithm appeared to be the best in all models, and at sample sizes of n = 5, 15, 30, and 80.
2. We also note that the MLE appeared to be the best in the first three models, and at sample sizes of n = 150.
3. The genetic algorithm appeared to be the best in the last model, and at sample sizes of n = 150.

Figure No. (3): The following graphs represent the estimators of the Risk function of the EKIW distribution for different values of the parameters.

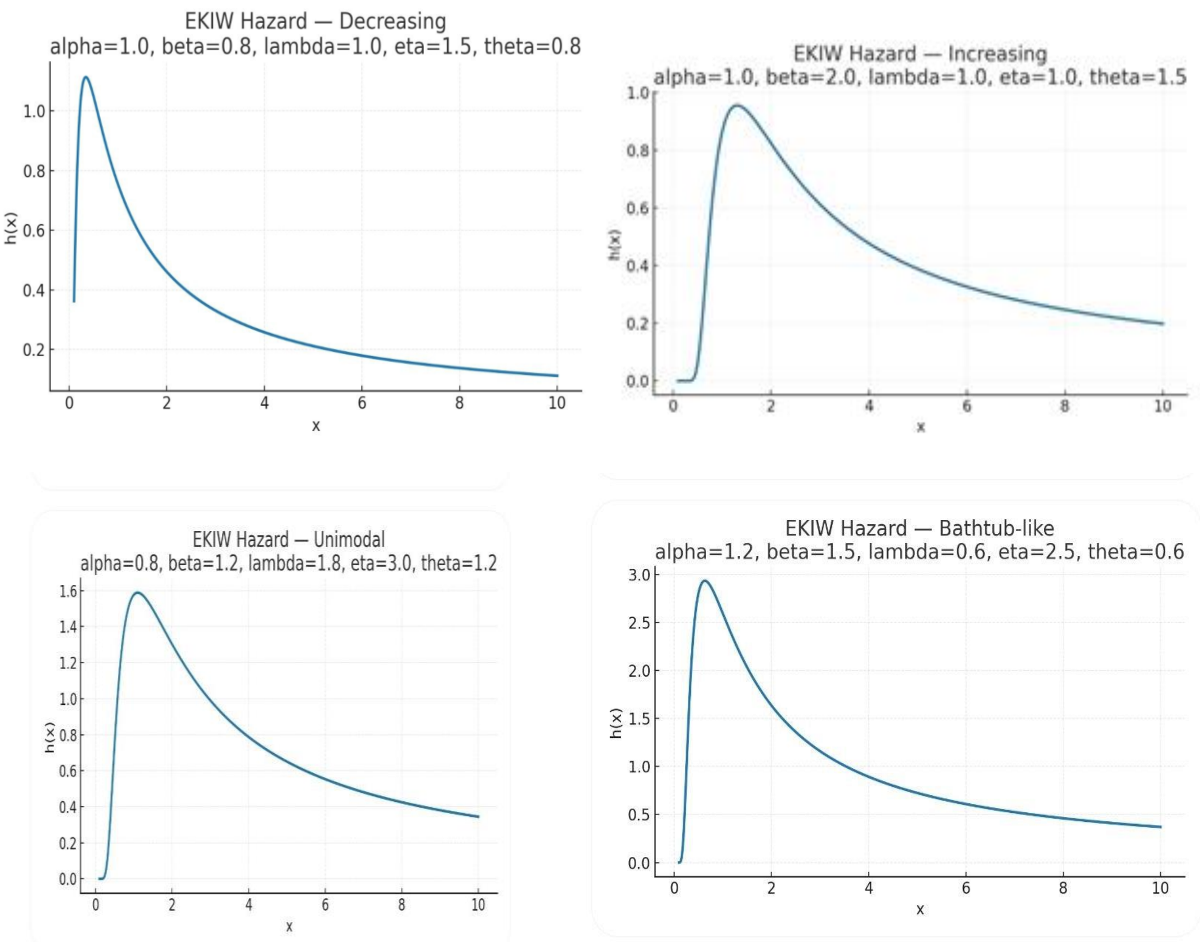

Figure 4. Graphs represent the estimators of the Risk function of the EKIW distribution for different values of the parameters.

Practical Aspect

The study included complete data on 150 lung cancer deaths, collected from the National Center for Cancer Research at the University of Baghdad during 2025. The data represented the period from the patient's admission to the center until death.

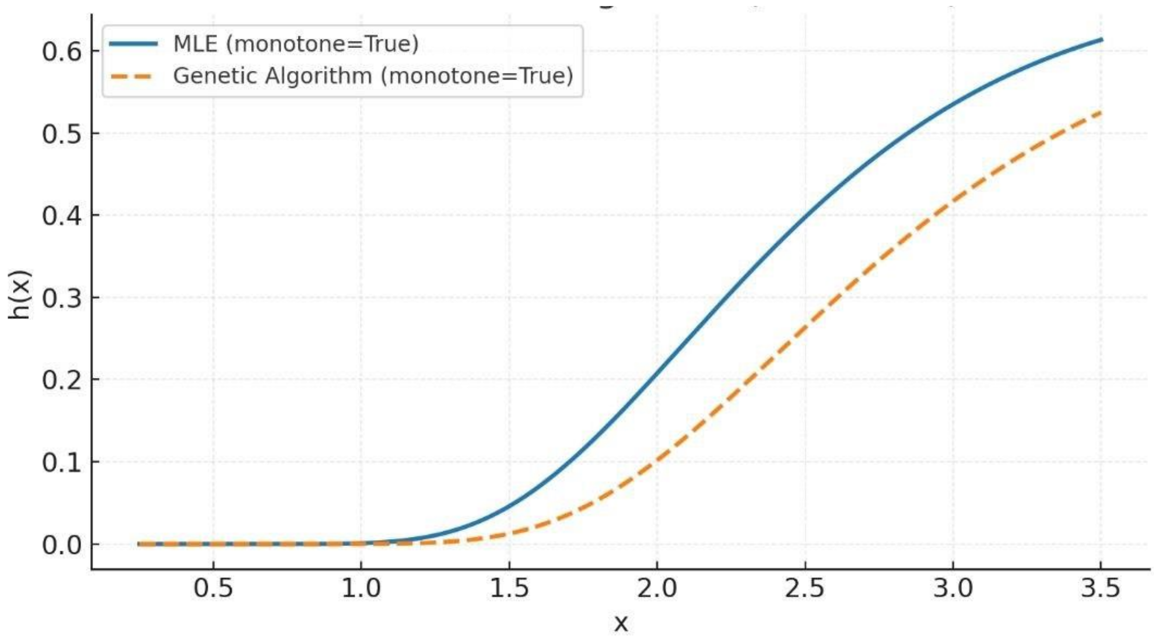

Figure 5. Represents the capabilities of the risk function for the MLE and the genetic algorithm method

We note from Figure (1) that the behavior of the risk function estimates for the two methods appeared increasingly for the EKIW distribution.

Figure No. (6): Histogram and estimated densities

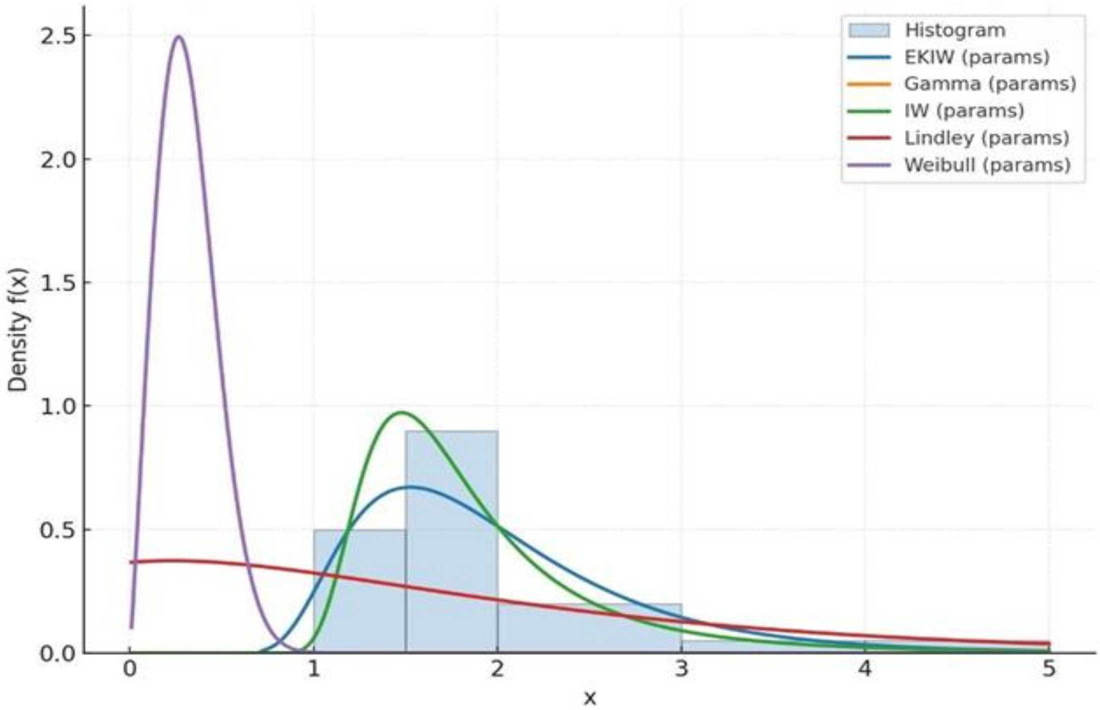

Figure 6. Histogram and estimated densities

Figure No. (6): The graphs of the estimated density functions demonstrate that the EKIW distribution represents the data more efficiently than the Gamma, Inverse Weibull, Lindley, and Weibull distributions, demonstrating its superior model fit to these data.

Conclusions

Through the results of the simulation and application aspects, we reached the following conclusions:

1. The genetic algorithm performed best in all models, at sample sizes of n = 5, 15, 30, and 80.
2. The maximum likelihood method performed best in the first three models, at sample sizes of n = 150.
3. The genetic algorithm performed best in the last model, at sample sizes of n = 150.
4. The hazard function showed an increasing value with increasing duration of disease in the group of lung cancer patients studied. This is consistent with the theoretical properties of this function, which is a monotonic increasing function.

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